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# PROCEEDINGS

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# DISCUSSION OF RIVER CHANNEL ROUGHNESS

(*Published in July, 1951*)

By T. Blench; James J. Doland and Ven Te Chow; Robert B. Banks; L. Bajorunas; and Sir Claude Inglis

HYDRAULICS DIVISION



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# DISCUSSION

T. Blench,[10] M. ASCE.—The extension of Dr. Einstein's methods[9] to deduce the gauge-discharge relation of a regime-type[11,12] river seems over optimistic. The following phenomena, that seem to be well established, illustrate the reasons for the writer's belief:

1. In laboratory flume experiments[13] with a bed of natural regime-type river sand, constant discharge, and various bed-sand injection rates, observations after steady regime depths had been attained showed that:

*a.* Rugosity was greatly altered by changes of injection rate within the practical large-river range;

*b.* Bed-wave full amplitude was on the order of one-fifth water depth;

*c.* After a moderate displacement from regime, a 30-ft flume, carrying about one-fifth cu ft per sec, requires some 80 hr to settle down to regime;

*d.* Bed-wave conformation changes markedly during attainment of regime, and bed waves have greatly enhanced amplitude when degradation is occurring.

2. In a large regime-type river[14] daily sonic observations of bed waves during a month including a high flood peak showed that:

*a.* Bed-wave length is closely related to discharge;

*b.* Bed-wave full amplitude is up to 40% water depth at high discharge.

*c.* Mean size of bed material in a reach between diversions is very closely correlated with estimated formative depth. (Samples were taken daily on various verticals in which formative depth did not vary appreciably with time.)

*d.* Discharge division between two branches is found to agree reasonably with the regime theory slope formula in terms of bed-sediment size.

3. In the same river it was found that distribution of sand sizes within bed samples follow a simple universal law.

4. In river models generally[15] it has been found that successful models have width and depth scales related in consonance with regime theory derived from the study of self-formed, sediment-bearing canals of controlled discharges.

In terms of the preceding facts the following phenomena might be expected in rivers: (*a*) Rugosity dependent on rate of change of discharge, that is, on hydrograph form; (*b*) rugosity dependent on bed-load discharge intensity,

Note.—This paper by Hans A. Einstein and Nicholas L. Barbarossa was published in July, 1951, as *Proceedings-Separate No. 78*. The numbering of footnotes, illustrations, tables, and equations in this Separate is a continuation of the consecutive numbering used in the original paper.

[10] Associate Prof. of Civ. Eng., Univ. of Alberta, Edmonton, Alta., Canada.

[9] "Formulas for the Transportation of Bed Load," by H. A. Einstein, *Transactions*, ASCE, Vol. 107, 1942, p. 561.

[11] "Regime Theory for Self-Formed Sediment-Bearing Channels," by Thomas Blench, *Proceedings-Separate No. 70*, ASCE, May, 1951.

[12] "Hydraulics of Sediment-Bearing Canals and Rivers," by Thomas Blench, Evans Industries Ltd., Vancouver, B. C., Canada, 1951.

[13] "Experiments for Fraser River Model," National Research Council of Canada, 1950–51, Fraser River Model Office, Univ. of British Columbia, Vancouver, B. C., Canada (unpublished).

[14] "Final Report on Special Observation at Ladner Reach during 1950 Freshet," by E. S. Pretious and T. Blench, National Research Council of Canada, 1951.

[15] "Scales for Moving-Bed River Models," by Thomas Blench, London Dock and Harbour Authority, London, England, March, 1951.

which can be affected by conditions upstream; (*c*) rugosity affected by meander pattern; (*d*) at some average discharge, conditions explicable in terms of regime theory for steady discharge; (*e*) Eq. 6, which pertains to noncorrugated boundaries, will be inapplicable except after severe distortion of its "constant"; and (*f*) for the conditions at which regime theory applies, only one parameter should be needed to define the bed-sediment size.

Therefore, the points on Fig. 3 should be expected to scatter about a mean line representing regime conditions. The coordinates are nondimensional numbers containing the principal pertinent variables with the exception of bed-load discharge intensity. The line cannot be expected to have a simple equation since it has to correct for the misuse of Eq. 6; the two values of $D$ are chosen in an arbitrary manner; and the use of any kind of $D$-term presumably hides the settlement velocity that does not follow a simple law in the size range of river sands.

Fig. 3 is of considerable value to the writer. It shows that, by choosing suitable river sites and analyzing data scientifically, a regime-type relation is demonstrated, just as such relations were shown by the much simpler data of controlled canals. Considering the scatter of points in canal analyses when regime theory was being established, the scatter on Fig. 3 is quite small. But, if instead of using the points of Fig. 3 to establish the existence of a regime relation, the smoothing curve is used to indicate where the point for a given stage of a given river is likely to be, then the scatter is very bad; for example, where the curve indicates a value of $V/V''_f$ of 10 the correct answer appears to be anywhere between about 8 and 14 depending on a host of circumstances of which there is no measure.

The writer emphasizes that his criticism is of an application; he holds Dr. Einstein's basic work, from which the present paper springs, in the highest regard and has used it in practical applications.

James J. Doland,[16] M. ASCE, and Ven Te Chow,[17] A. M. ASCE.—In applying the Manning formula to a natural stream, engineers as well as engineering students often ignore two important facts:

1. In natural streams the flow is rarely uniform because of the effect of the inevitable change in discharge due to the varying shape and size of the channel cross section and the unavoidable irregularities in channel alinement;

2. In natural streams the roughness value $n$ is not constant, varying with the discharge and the depth of water in the channel. This can be best illustrated by a typical case, such as is shown in Fig. 6, indicating how the roughness varies with the depth of water in the channel and with the corresponding variation in discharge. The $n$-value is high at high stages because of rough and grassy banks and increases rapidly near the bottom where debris has accumulated.

Engineers and students usually learn the Manning formula from college hydraulics, in which the theoretical cases are treated (for which cases the formula

[16] Prof., Hydr. Eng., Univ. of Illinois, Urbana, Ill.

[17] Research Asst., Prof. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

is designed only for uniform flow and a single-valued $n$ is generally assigned), so it often happens that in practical problems they apply the Manning formula indiscriminately to natural streams in the same way as to theoretical or artificial channels. The condition of uniform flow and constant roughness is unconsciously assumed without apparent reason. The writers are glad that the paper

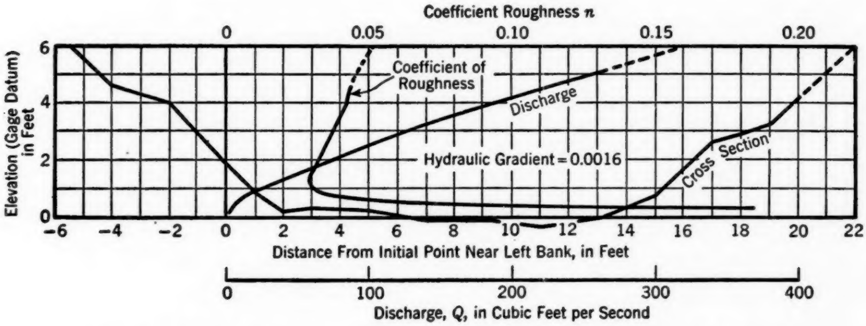


FIG. 6.—HYDRAULIC CHARACTERISTICS OF BONEYARD CREEK AT GAGING STATION, UNIVERSITY OF ILLINOIS, URBANA, ILL.

points out that in the hydraulic computations for roughness in natural channels, the assumption of uniform flow may be made with negligible error. In practical problems the variation of $n$ with $Q$ is so evident that there is a need to develop a new and more rational procedure for estimating the varying roughness in natural channels.

It seems that the procedure of hydraulic computation described in the paper may be simplified with practical advantages so that the most familiar form of the Manning formula is still retained in use and that several terms unfamiliar to practicing engineers are eliminated.

*Relationship Between* $\left(\frac{R}{K_s}\right)^{\frac{1}{3}}$ *and* $\frac{D}{R\,S}$ *for Given Values of* $\frac{R'}{R}$.—In the paper the definition of $V'_f$ and $V''_f$ and the equation for $V/V'_f$ are given as follows:

$$V'_f = (g\,S\,R')^{\frac{1}{2}} \qquad (20)$$

$$V''_f = (g\,S\,R'')^{\frac{1}{2}} \qquad (21a)$$

and

$$\frac{V}{V'_f} = 7.66\left(\frac{R'}{K_s}\right)^{1/6} \qquad (21b)$$

Eliminating $V'_f$ and $S$ from the equations and simplifying,

$$\frac{R''}{R'} = \frac{58.68\left(\frac{R'}{K_s}\right)^{\frac{1}{3}}}{\left(\frac{V}{V''_f}\right)^2} \qquad (22)$$

Introducing $R'' = R - R'$ in Eq. 22 and solving,

$$\left(\frac{R}{K_s}\right)^{\frac{1}{3}} = \frac{1}{58.68} \frac{\left(\frac{V}{V''_f}\right)^2 \left[1 - \left(\frac{R'}{R}\right)\right]}{\left(\frac{R'}{R}\right)^{4/3}} \quad \text{.............(23)}$$

From Eq. 17,

$$\frac{D}{R\,S} = \frac{\psi'}{1.68}\,\frac{R'}{R} \quad \text{.......................(24)}$$

Now, by assuming various values of $R'/R$, Eqs. 23 and 24 will give the relationships between $\left(\frac{R}{K_s}\right)^{\frac{1}{3}}$ and $\frac{V}{V''_f}$ and between $\frac{D}{R\,S}$ and $\psi'$, respectively. As the relationship between $V/V''_f$ and $\psi'$ is shown by the curve in Fig. 3 the relationship between $\left(\frac{R}{K_s}\right)^{\frac{1}{3}}$ and $\frac{D}{R\,S}$ can be obtained indirectly. For example, assume $\frac{R'}{R} = 0.60$, then, from Eqs. 23 and 24, $\left(\frac{R}{K_s}\right)^{\frac{1}{3}} = \frac{1}{73.8}\left(\frac{V}{V''_f}\right)^2$ and $\frac{D}{R\,S} = \frac{\psi'}{2.80}$. Take $\psi' = 5.60$. Then, from Fig. 3, $\frac{V}{V''_f} = 11.5$. The corresponding values of $\left(\frac{R}{K_s}\right)^{\frac{1}{3}} = 1.79$ and $\frac{D}{R\,S} = 2.00$. The writers have computed a number of pairs of the values of the item $\left(\frac{R}{K_s}\right)^{\frac{1}{3}}$ and $\frac{D}{R\,S}$ for various values of $R'/R$. The results are plotted in the form of curves as shown in Fig. 7.

*Equation for Roughness Factor n.*—Since the Manning formula is

$$V = \frac{1.486}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \quad \text{.......................(3)}$$

and Eq. 6 gives

$$V = 7.66\, V'_f \left(\frac{R'}{K_f}\right)^{1/6} \quad \text{.....................(25)}$$

in which

$$V'_f = (g\,S\,R')^{\frac{1}{2}} \quad \text{.......................(20)}$$

The roughness factor $n$ may be then expressed by eliminating $V$, $V'_f$, and $S$ from Eqs. 3, 20, and 25:

$$n = \frac{\frac{K_s^{1/6}}{29.2}}{\left(\frac{R'}{R}\right)^{\frac{2}{3}}} \quad \text{.......................(26)}$$

*Procedure of Hydraulic Computation.*—By means of the curves in Fig. 7 and Eq. 26 the practical procedure for computing the rating curve may be performed as follows:

*a.* From a given data for $y$, $S$, $K_s$, and $D$, compute $R$, $\left(\frac{R}{K_s}\right)^{\frac{1}{3}}$, and $\frac{D}{R\,S}$;

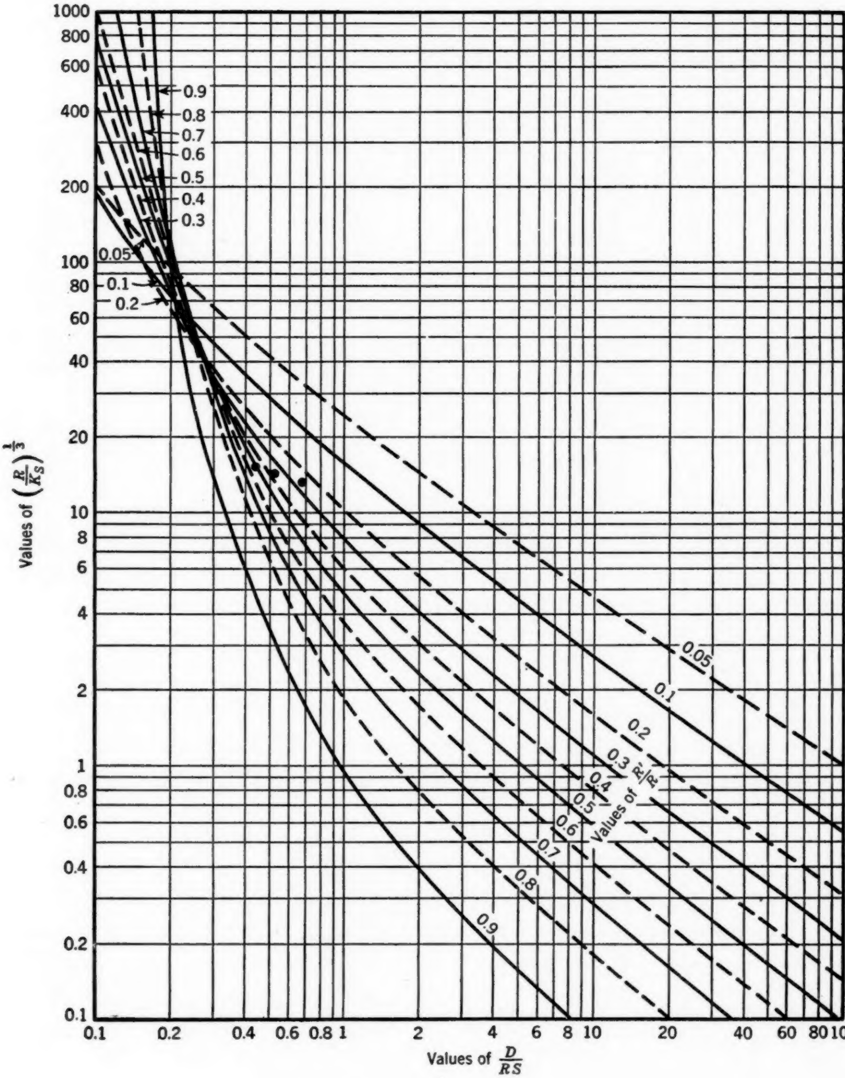


FIG. 7.—RELATIONSHIP CURVES

*b.* Using the computed values of $\left(\frac{R}{K_s}\right)^{\frac{1}{6}}$ and $\frac{D}{R\,S}$, find the value of $\frac{R'}{R}$ from Fig. 7;

*c.* Compute $n$ by Eq. 26, using the given $K_s$ and the value of $R'/R$ obtained in step 2; and

*d.* Compute the discharge by the Manning formula.

TABLE 3.—Typical Computations for Rating Curve

| $y$ | $R$ | $\left(\frac{R}{K_s}\right)^{\frac{1}{6}}$ | $\frac{D}{R\,S}$ | $\frac{R'}{R}$ | $n$ | $A$ | $Q$ |
|---|---|---|---|---|---|---|---|
| 6.7 | 3.81 | 13.24 | 0.688 | 0.260 | 0.0286 | 1,010 | 2,860 |
| 8.1 | 5.00 | 14.50 | 0.524 | 0.366 | 0.0229 | 1,380 | 5,850 |
| 9.0 | 5.85 | 15.25 | 0.448 | 0.513 | 0.0178 | 1,649 | 9,630 |

Using the authors' example for illustration, the procedure is: (1) Assume $y = 6.7$ ft., $S = 5.00 \times 10^{-4}$, $K_s = 16.4 \times 10^{-4}$, and $D = 13.1 \times 10^{-4}$; then $R = 3.81$, $\left(\frac{R}{K_s}\right)^{\frac{1}{6}} = 13.24$, and $\frac{D}{R\,S} = 0.688$; (2) from Fig. 7, $\frac{R'}{R} = 0.26$; (3) by Eq. 26, $n = 0.0286$; and (4) using the Manning formula, $Q = 2{,}860$ cu ft per sec. The complete computation is shown in Table 3. The corresponding points involved in step 2 are indicated by dots in Fig. 7.

It is clear that the idea presented in this paper is developed into a procedure that is practicable and workable. However, the accuracy of the procedure depends on many factors. The two most important ones are:

(*a*) Satisfaction with the use of $K_s$ to describe the grain roughness. According to the theory of statistics the distribution of an observed variate may be described by a number of parameters. In engineering works, parameters greater than the third order (skewness) are usually insignificant, and hence rarely employed. However, on the other hand, a single parameter is too in-

TABLE 4.—Range of Variation in Computation of Roughness Factor $n$

| Value of $R'$ | Depth y, in Feet | | | Discharge $Q$, in Cubic Feet per Second | | | Roughness Factor $n$ | | | Percentage variation |
|---|---|---|---|---|---|---|---|---|---|---|
| | Maximum | Minimum | Curve | Maximum | Minimum | Curve | Maximum | Minimum | Curve | |
| 1.00 | 8.25 | 5.40 | 6.70 | 4,020 | 1,415 | 2,860 | 0.044 | 0.023 | 0.029 | 72.4 |
| 2.00 | 9.60 | 7.00 | 8.10 | 8,160 | 5,030 | 6,200 | 0.025 | 0.019 | 0.023 | 26.1 |
| 3.00 | 10.00 | 8.10 | 9.00 | 11,600 | 8,120 | 9,630 | 0.020 | 0.017 | 0.018 | 20.0 |

definite to describe a distribution. It seems that the distribution of grain size is so irregular that it cannot be described completely by any single value of representative diameter unless the pattern of distribution is of the same type and shape for all possible cases.

(*b*) The accuracy of the theory upon which the curve of Fig. 3 is based. The authors have obtained the working relationship between the values of $V/V''_f$ and $\psi'$ from actual measurements for a number of rivers. This relationship is represented by a smooth curve fitted to the observed data. However, it appears that the plotted observed points scatter within a band of certain

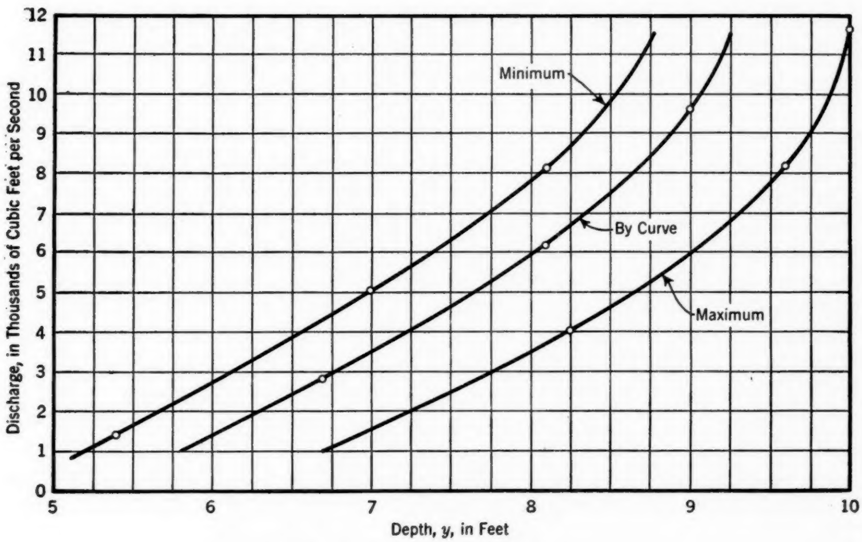


FIG. 8.—VARIATION IN RATING CURVES

width rather than along a narrow determinable path that could be well represented by a line. It is interesting to see what the effect of this scatter is on the computed roughness factor *n*. Table 4 gives a computation in which the extreme values taken from the band of the plotted points in Fig. 3 are used. The range of variation in *n* amounts to as high as 72.4%. The resulting rating curves are shown in Fig. 8.

ROBERT B. BANKS,[18] J. M. ASCE.—With rare exception, the historical problem of hydraulic roughness has been concerned with the development of empirical expressions for determining fluid resistance. However, in the case in which one type of roughness may be considered to be superimposed over another type, empirical formulas expressing the friction loss of the total roughness either do not exist, as in the authors' problem, or the formulas introduce such vague characteristics as an "equivalent sand roughness." Consequently, as the authors remark, the determination of friction losses in this case is necessarily based entirely upon judgment. Present-day knowledge of the mechanics of fluid flow should certainly warrant more precise statements "* * * a tortuous river with considerable vegetation possesses a Manning coefficient of 0.04." The rational approach proposed by the authors to the

[18] Fulbright Fellow, Imperial College of Science and Technology, London, England.

problem of river channel roughness thus permits the engineer to calculate the Manning coefficient for this kind of "composite" roughness.

The writer, in conjunction with Mr. Einstein, conducted an experimental investigation concerned with the resistance exerted by a boundary composed of two or more types of friction-producing elements acting over the same bed area.[19,20] Results obtained from this investigation have led to the conclusion that other types of hydraulic problems also may be solved by utilizing the concept given by the authors. For example, it is possible to calculate the Chezy coefficient for a particular flow for the case in which small sills are attached, widely separated, to a flow boundary. In this case, one assumes that the total resistance to the flow is equal to the sum of the resistances exerted by the sills, and by the boundary area between the sills. Consequently, if expressions are available which give the resistance exerted by each type of roughness, the total resistance can be obtained directly, and from the latter the Chezy coefficient is computed.

One might consider the authors' parameter, $\psi'$, as a ratio of weight to shear. Multiplying the numerator and the denominator of Eq. 17 by $g$, it is seen that the former becomes the submerged weight of a layer $D_{35}$ in height, and that the latter becomes the grain shear. Suppose that a flow is increased; then the rate of bed load transport increases and, accordingly, $\psi'$ decreases. For $\psi'$ to decrease, the denominator of Eq. 17 must increase, which is compatible with the assumption regarding an increased flow.

It is to be hoped that river measurements taken by others in the future can be calculated in terms of the ordinates of Fig. 3 and included with the authors' data. Meanwhile, by employing the curve which the authors regard as tentative and by utilizing their concept of resolution of shears, the problem of predicting river channel friction losses can be solved.

L. BAJORUNAS[21].—Available "open-channel flow" formulas do not permit accurate computation of discharges for varying stages. There are two ways of adjusting such formulas to produce accurate results. One is to adjust the roughness factor with changing stage and the other is to change the river cross section. Since no rules have existed for making such adjustments, the proposal in this paper will be very helpful.

The writer concurs with the authors (discussion preceding Eqs. 18) that, for practical purposes, the Manning-Strickler equation (Eq. 6) is more convenient to use than Eq. 7 or Eq. 9, because solutions are direct. However, if it is desired to use Eq. 6 in a practical application, the data used to develop the relationship (as the curve in Fig. 3) should be computed by use of Eq. 6 and not by Eqs. 7, 8, and 9, as was done by the authors.

The writer suggests the following procedure to evaluate the necessary relationships for the Manning equation: Assume that the Manning roughness

[19] "Fluid Resistance of Composite Roughness," by H. A. Einstein and R. B. Banks, *Transactions*, Am. Geophysical Union, August, 1950, pp. 603–610.

[20] "Linearity of Friction in Open Channel Flow," by R. B. Banks, dissertation presented to the Univ. of California, at Berkeley, Calif., in June, 1951, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[21] Hydr. Engr., Garrison (N. Dak.) District, Corps of Engrs., U. S. Dept. of the Army.

factor is composed of two parts:

$$n = n' + n'' \qquad (27)$$

in which

$$n' = \frac{K^{1/6}_s}{29.3} \qquad (28)$$

is a roughness factor depending on channel bed grain diameter and $n''$ is the roughness factor depending on channel bed irregularities.

The Manning equation is expressed by Eq. 3 and the Manning-Strickler equation is

$$V = \frac{1.486}{n'} R'^{2/3} S^{1/2} \qquad (29)$$

which is identical with Eq. 6. The expression for $n$ given in Eq. 4 is properly an expression for $n'$, the grain roughness, and the roughness factor $n$ in Eq. 3 may be replaced by $K^{1/6}_s/29.3$ only if $R$ is replaced by $R'$. It follows from Eqs. 3 and 29 that

$$R = R' \left(1 + \frac{n''}{n'}\right)^{3/2} \qquad (30a)$$

or

$$R = R' \left(\frac{n}{n'}\right)^{3/2} \qquad (30b)$$

By reasoning similar to that set forth by the authors in the discussion preceding Eq. 17, $n''$ is assumed to be a function of the sediment transport $\psi'$. To determine the relationship between $n''$ and $\psi'$ the data from actual river flow measurements were used—namely, all data on which Fig. 3 is based sup-

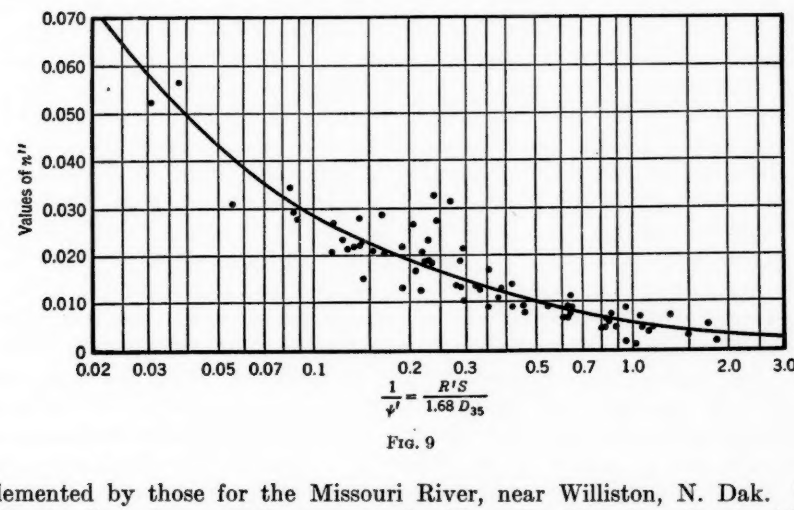


Fig. 9

plemented by those for the Missouri River, near Williston, N. Dak. The following line of computations was used: $n'$ from Eq. 28, $n$ from Eq. 3, with $V$, $R$, and $S$ from flow measurements, $n''$ from Eq. 27, $R'$ from Eq. 30, and $\psi'$ from Eq. 17. No adjustments were made for friction losses on banks.

The results are indicated in Fig. 9, with $n''$ as the ordinate and $1/\psi'$ as the abscissa, indicating the increase of sediment transport from left to right in the same manner as that proposed by Carl B. Brown,[22] M. ASCE. In Fig. 9 the roughness factor that reflects the channel irregularities $n''$ decreases with increasing flow and has a minimum value approaching zero.

The procedure for a practical application could be the following: Compute $n'$ from Eq. 28; assume $R'$; determine $1/\psi'$ from Eq. 17; read off $n''$ from Fig. 9; and calculate $n$ from Eq. 27, $R$ from Eq. 30, and $V$ from Eq. 3. For repeated use, it is advisable to draw a curve with $n$ as ordinate and $R$ as abscissa.

*Acknowledgment.*—The co-operation of Mr. Einstein in providing the flow data used in constructing Fig. 3, and of H. K. Pratt, M. ASCE (Chief, Reports and Hydraulics Branch of Garrison District of the Corps of Engineers), for suggestions and for permitting the use of the data for the Missouri River, near Williston, is gratefully acknowledged.

Sir Claude Inglis[23] M. ASCE.—The physical approach to the subject of channel roughness and its relationship to sand transport is sound as long as the authors restrict it to two-dimensional flow; but it is an over-simplification to assume that all energy losses except those due to grain roughness were caused by shear force resulting from irregularities of the periphery. In a laboratory investigation of energy losses in one-way (river) and two-way (estuary) flow, at the Hydraulics Research Station of the Department of Scientific and Industrial Research at Wallingford, Berkshire, England, the energy losses have been divided into three categories: (1) Those due to textural roughness; (2) those due to ripple roughness; and (3) those caused by form drag resulting from the major irregularities of the banks, bed, islands, and sandbanks—all of which induce curvature of flow and tend to produce meanders. The authors cannot justifiably group these losses with irregularities of the bed due to sand ripples, bars, and sand dunes because, whereas the bed pattern and bed movements adjust themselves fairly rapidly as the discharge varies, this does not occur in the case of major irregularities such as promontories, outcrops, or gradually changing banks, islands, and sandbanks, which have an entirely different time scale from that of bed pattern adjustment by the movement of sand grains.

Next, as regards the size of grains exposed on the bed—the writer cannot agree that the size remains constant and that the 35:65 percentage distribution of material remains the same at all discharges. In India, the writer found material in suspension, material exposed on the bed, and also bed levels to vary considerably with different discharges. To collect samples of bed material at various discharges, on the other hand, would make the work very laborious.

It also appears that the authors have assumed not merely the same drop between the upstream and downstream observation points but also the same uniform slope for all discharges. Actually, however (as the authors themselves state), the curvature of flow and, hence, the distance the water travels, decreases as the discharge increases. In other words, the slope increases; and in

[22] "Engineering Hydraulics," edited by H. Rouse, John Willey & Sons, Inc., New York, N. Y., 1950, p. 796.

[23] Director, Hydraulics Research Station, Dept. of Scientific Industrial Research, Wallingford, Berkshire, England.

many rivers the slope is far from uniform, at lower stages consisting of a series of pools and rapids.

Rather than a rational approach, the writer considers that linking the flow with a grain size which is assumed to be unaffected by discharge, as advanced in this paper, is extreme empiricism, in which variables are assumed to be constants.

The next question concerns the degree of accuracy attained. E. S. Crump (senior hydraulic engineer on the writer's staff) has analyzed the data. Beginning with the first item of Table 1 (Missouri River, Pierre, S. Dak.), he found by trial and error the results shown in Table 5. The labor involved in

TABLE 5.—ANALYSIS OF MISSOURI RIVER DATA (PIERRE, S. DAK.; TABLE 1) BY E. S. CRUMP

| Stage | $R'$ | | $R''$ | | $R' + R''$ | | $R''/R'$ | | $Q$ | | $\frac{Q_c}{Q_m}$ | Error |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Computed | Measured | Computed | Measured | Computed | Measured | Computed | Measured | Computed, $Q_c$ | Measured, $Q_m$ | (%) | (%) |
| Low.... | 0.300 | 0.880 | 2.960 | 2.380 | 3.26 | 3.26 | 9.87 | 2.70 | 2,616 | 5,200 | 50 | −50 |
| High.... | 5.103 | 6.730 | 3.687 | 2.060 | 8.79 | 8.79 | 0.72 | 0.31 | 128,700 | 145,000 | 89 | −11 |

obtaining these results was so great that Mr. Crump adopted a simpler method for obtaining the probable error, which had the added advantage that it represented average performance of the several channels in the same manner as does the standard line in Fig. 3 which the authors recommend for solving specific problems. Disregarding results for the Salinas River at Paso Robles, Calif., the scatter of results has a standard deviation of $\pm$ 20% (very approximately) in the ordinate $\phi'' = V/V''_f = V/\sqrt{g\,S\,R''}$ for a given value of the abscissa $\psi' = \frac{(1.68\,D_{35})}{(S\,R')}$ of Fig. 3. For a given site where the constants $K_s$, $D_{35}$, and $S$ are known, and the values of $A$ and $R$ for various values of the gage $y$ have been extablished by survey, the authors' method of calibrating the gage for discharge $Q = V\,A$ is to assume successive values of the component $R'$. For each value assumed, the velocity $V$ is determined from Eq. 9; and for the known value of $\psi'$ = constant $\div R'$, the corresponding value of $R'' = \frac{V^2}{g\,S}\cdot\left(\frac{1}{\phi''}\right)^2$ is found by reading the value of $\phi''$ from the curve of Fig. 3. Calling $R''_E$ the value so obtained, it is seen that with a standard deviation in $\phi''$ of $\pm$ 20% values of $R''$ may range from $R''_{0.8} = \left(\frac{1}{0.80}\right)^2 R''_E = 1.562\,R''_E$ to $R''_{1.2} = \left(\frac{1}{1.2}\right)^2 R''_E = 0.694\,R''_E$. The effect of this range of error on $R = R' + R''$ obviously depends on the ratio of $R''$ to $R$ or $R'$.

Following this method Mr. Crump found that the average value of $R''/R$ was 5.80 for low discharges, 2.00 for intermediate discharges, and 0.66 for high discharges; and, accepting the foregoing average values of $R''/R$ as typical, he found that the effect on $R = R' + R''$ of standard error in $\phi''$ were as shown in

Table 6. The effect of these standard deviations on values of $R$ is shown by the ratios in Cols. 8 and 9. The difficulty of expressing the effect in terms of relative discharges will be apparent from Fig. 10 which indicates the relationship

TABLE 6.—ANALYSIS OF THE EFFECT OF STANDARD ERROR IN $\phi''$

| Stage of flow | $\frac{R''_E}{R'}$ | $\frac{R_E}{R'}$ | $\frac{R''_{0.8}}{R''_E}$ | $\frac{R''_{1.2}}{R''_E}$ | $\frac{R_{0.8}}{R'}$ | $\frac{R_{1.2}}{R'}$ | $\frac{R_{0.8}}{R_E}$ | $\frac{R_{1.2}}{R_E}$ |
|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Low | 5.80 | 6.80 | 1.562 | 0.694 | 10.062 | 5.025 | 1.480 | 0.739 |
| Medium | 2.00 | 3.00 | 1.562 | 0.694 | 4.124 | 2.388 | 1.375 | 0.796 |
| High | 0.658 | 1.658 | 1.562 | 0.694 | 2.028 | 1.456 | 1.223 | 0.877 |

Explanation:
In the headings, subscript $E$ denotes values given by the standard curve, Fig. 3, and subscripts 0.8 and 1.2 denote values that are, respectively, 20% less than, and 20% greater than, those given in Fig. 3.

Col. 3 = 1.0 + Col. 2
Col. 6 = 1.0 + Col. 2 × Col. 4
Col. 7 = 1.0 + Col. 2 × Col. 5
Col. 8 = Col. 6 ÷ Col. 3
Col. 9 = Col. 7 ÷ Col. 3

between $V$ and $R$ for three values of $\phi''/\phi''_E$—namely, 0.8, 1.0, and 1.2, respectively. The curves are drawn for the case of medium flow, for which the abscissas of such points as C, B, and A have a constant ratio of 1.375:1.000:0.796. Being based on a common value of the component $R'$, all such points have the same value of $V$ and therefore lie on a horizontal line. It is required

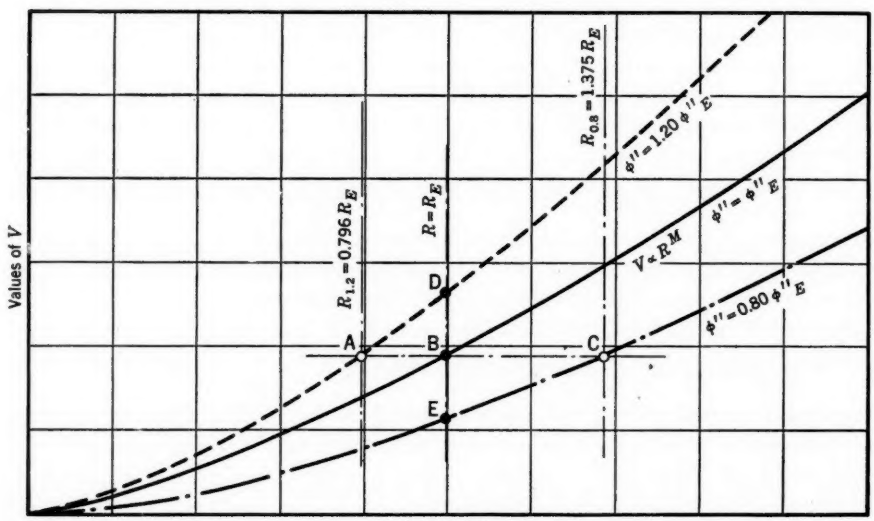


FIG. 10.—AVERAGE RELATIONSHIP BETWEEN $V$ AND $R$ FOR AUTHORS' DATA

to know the ratio of the ordinates of points D, B, and E which lie on the same vertical and therefore have the same value of $R$, $y$ (gage reading) and $A$ (sectional area) so that the ratio of the discharges $Q_D:Q_B:Q_E$ is the same as the ratio

of the velocities $V_D : V_B : V_E$. To know these ratios it is necessary to know the value of the index $m$ in the assumed relationship $V \alpha R^m$. Fortunately, in Table 1 the authors have given two values of $V$ and $R$ in seven of the nine sites taken into consideration. With $\frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^m$ the value of $m$ is given by $m = \log \frac{V_1}{V_2} \div \log \frac{R_1}{R_2}$. Similarly, if the ratio of the roughness factors, $\frac{N_2}{N_1} = \left(\frac{R_1}{R_2}\right)^n$, the value of the index $n$ is given by $n = \log \frac{N_2}{N_1} \div \log \frac{R_1}{R_2}$.

TABLE 7.—COMPUTATION OF $m$ AND $n$ FROM DATA IN TABLE 1

| Legend Fig. 3 | $R_1$ | $R_2$ | $V_1$ | $V_2$ | $N_1$ | $N_2$ | $\frac{R_1}{R_2}$ | $\frac{V_1}{V_2}$ | $\frac{N_2}{N_1}$ | $m$ | $n$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi$ | 8.79 | 3.26 | 5.50 | 1.64 | 0.015 | 0.026 | 2.698 | 3.352 | 1.736 | 1.218 | 0.556 |
| 0 | 11.13 | 4.53 | 6.12 | 1.11 | 0.014 | 0.045 | 2.460 | 5.510 | 3.216 | 1.895 | 1.298 |
| $\Delta$ | 6.62 | 1.52 | 7.67 | 1.32 | 0.016 | 0.034 | 4.355 | 5.810 | 2.126 | 1.160 | 0.512 |
| + | 13.40 | 3.64 | 3.46 | 1.14 | 0.033 | 0.042 | 3.680 | 3.034 | 1.273 | 0.852 | 0.185 |
| × | 5.54 | 1.68 | 9.14 | 1.64 | 0.014 | 0.034 | 3.298 | 5.575 | 2.434 | 1.440 | 0.740 |
| $\nabla$ | 3.18 | 0.69 | 6.67 | 0.61 | 0.016 | 0.065 | 4.610 | 10.940 | 4.060 | 1.566 | 0.918 |
| □ | 4.59 | 1.27 | 7.72 | 0.84 | 0.019 | 0.076 | 3.616 | 9.190 | 4.000 | 1.724 | 1.079 |
| Average | .... | .... | .... | .... | .... | .... | .... | .... | .... | 1.408 | 0.755 |

Computed values of $m$ and $n$ for the seven cases considered are given in Table 7. For the authors' results, Table 7, these computations establish the average relationship,

$$V \alpha R^{1.41} S^{0.5} \qquad (31)$$

Comparison with Manning's formula—

$$V \alpha \frac{R^{0.67}}{N} S^{0.5} \qquad (32)$$

—requires that

$$\frac{R^{0.67}}{N} \alpha R^{1.41} \qquad (33a)$$

or

$$\frac{1}{N} \alpha R^{0.74} \qquad (33b)$$

Eq. 33*b* yields $n = 0.74$ as against $n = 0.755$ found by analysis.

Reverting to Fig. 10, with $V \alpha R^{1.41}$, it is now possible to find the ratio of the velocity ordinates (which is also the ratio of relative discharges) of the three points E, D, and B having the same sectional area. In moving along the lower curve from points C to B (Fig. 10) the abscissa $R$ is reduced in the ratio 1.375 : 1, so that the ordinate $V$ is reduced in the ratio $(1.375)^{1.41}$ : 1—that is, in the ratio 1 : 0.639. This is also the ratio of the discharges, $Q_{0.8} : Q_{1.0}$, the percentage error in discharge being $-36.1\%$. Similarly in moving from point A to point D along the upper curve, the ordinate $V$ is increased in the ratio $(0.796)^{1.41}$ : 1—that is, in the ratio 1 : 1.381; so that $Q_{1.2}/Q_{1.0} = 1.381$ giving an error of $+38.1\%$. This gives the result for medium stage flow. Results for the three stages of flow are shown in Table 8.

In the foregoing analysis, Mr. Crump has shown that the errors resulting from following the authors' method are far in excess of the ± 10% error to which experienced river engineers in India were able to work in assessing the discharges of selected short, straight reaches, and the claim (see sentence preceding "Acknowledgments") that "The obvious advantage of this method is that the results do not depend upon individual judgment for selection of the appropriate $n$ value"—is shown to be unjustified; because the errors due to the author's assumptions far exceed the errors to be anticipated in assessing the appropriate value for $n$ in a selected reach.

TABLE 8.—COMPARISON OF ERRORS IN COMPUTED VALUES OF $Q$, FROM FIG. 10

| Stage of flow | $\phi'' = 0.80\,\phi''_E$ | | | $\phi'' = 1.20\,\phi''_E$ | | |
|---|---|---|---|---|---|---|
| | $\frac{R_{0.8}}{R_E}$ | $\frac{Q_{0.8}}{Q_{1.0}} = \left(\frac{R_E}{R_{0.8}}\right)^{1.41}$ | Percentage error in $Q$ | $\frac{R_{1.2}}{R_E}$ | $\frac{Q_{1.2}}{Q_{1.0}} = \left(\frac{R_E}{R_{1.2}}\right)^{1.41}$ | Percentage error in $Q$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Low | 1.480 | 0.575 | −42.5 | 0.739 | 1.532 | +53.2 |
| Medium | 1.375 | 0.639 | −36.1 | 0.796 | 1.381 | +38.1 |
| High | 1.223 | 0.753 | −24.7 | 0.877 | 1.203 | +20.3 |